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Nonstatic global monopole in higher dimensional space time

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Abstract : We have obtained a class of nonstatic solutions around global monopole in higher dimensional space time. We have used a field theoretic energy momentum tensor for monopole configuration. This paper extends earlier work of Chakraborty (*Physica Scripta* **98** 294 (1998)) to its five dimensional analog.

Keywords : Nonstatic global monopole, higher dimension, deficit angles.

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1. Introduction

In last few years, there are attempts to unify gravity with other fundamental forces in nature. Latest studies of super string and super gravity theories and unification of fundamental forces with gravity reveal that the space-time dimension should be different from four [1]. As a result, higher dimensional theory is receiving great attention both in cosmology and in particle physics. Moreover, solutions of Einstein field equations in higher dimensional space-time are believed to be of physical relevance, possibly at the extremely early times before the Universe underwent compactification phase transitions. Phase transitions in the early Universe, can give rise to various forms of topological defects. A defect is a discontinuity in the vacuum and depending on the topology of the vacua, the defects could be domain walls, cosmic strings, monopoles and textures [2].

In particular, monopoles are formed when the vacuum manifold contains surfaces which cannot be continuously shrunk to a point, *i.e.* when $\pi_2(\mu) \neq 1$. Global monopoles (which are formed when a global symmetry is broken) are important objects both for particle physicists and cosmologists because it predicted to exist in Grand Unified theory.

The existence of a (magnetic) monopole was first suggested by Dirac [3]. Later, t' Hooft [4] and Polyakov [5] demonstrated monopole solutions in a gauge model possessing SO(3) symmetry with a Higgs field ϕ in a triplet representation. Subsequently, monopole solutions of the realistic grand unified models based on the gauge groups SU(5) and SO(10), have been considered by Dokos and Tomaras [6] and Li *et al* [7].

In 1989, Barriola and Vilenkin [8] have found an approximate solution of a monopole resulting from breaking of the global SO(3) of a triplet scalar field in a schwarzchild background.

An attractive feature of this topological defect is that it exerts no gravitational force on the matter around it (except for a tiny repulsive effect of the core). Some properties of the global monopole metric are similar to gauge cosmic strings [2] but an important difference is that the monopole metric is not locally flat. So far, most of the analytical works in monopoles deal with their gravitational effects based on general relativistic static models [9].

Recently, Chakraborty [10] described a non-static monopole solution. The consideration that the space time has more than four dimensions, has recently received much

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attention in its attempt to unify gravity with other gauge type interactions. As far our knowledge goes, there has not been any work in literature where nonstatic space-time has been considered for study of monopole in higher dimensional space-time. So, we have thought it worth while to extend the Chakraborty's work in five dimensional space-time.

2. The basic equation

In this section, we closely follow the formalism of Chakraborty and take the Lagrangion as [10]

$$L = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{4}\Lambda\left(\phi^a\phi^a - \eta^2\right)^2, \tag{1}$$

where ϕ^a is the triplet scalar field, $a = 1, 2, 3$ and η is the energy scale of the symmetry breaking and Λ is a constant.

The metric ansatz describing monopole can be taken as

$$ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + r^2H(t)$$
$$\times\left(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta\sin^2\phi d\psi^2\right). \tag{2}$$

The energy momentum tensor for the above Lagrangian is given by [10]

$$T_\mu^\nu = \nabla_\mu\phi^a\nabla^\nu\phi^a - L\delta_\mu^\nu. \tag{3}$$

Now for the above metric, the nonvanishing components of the energy momentum tensors are

$$T_t^t = -\frac{\dot\phi^{a^2}}{2A} - \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{4}\left(\phi^a\phi^a - \eta^2\right)^2, \tag{4}$$

$$T_r^r = \frac{\dot\phi^{a^2}}{2A} + \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{4}\left(\phi^a\phi^a - \eta^2\right)^2, \tag{5}$$

$$T_\theta^\theta = T_\phi^\phi = T_\psi^\psi = \frac{\dot\phi^{a^2}}{2A} - \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{2}\left(\phi^a\phi^a - \eta^2\right)^2, \tag{6}$$

$$T_t^r = \frac{1}{B}\dot\phi^a\phi'^a. \tag{6.1}$$

Thus, the Einstein equations are

$$\frac{3}{4}\frac{\dot B\dot H}{ABH} + \frac{3}{4}\frac{\dot H^2}{AH^2} + \frac{3}{2}\frac{B'}{B^2r} + \frac{3}{r^2H} - \frac{3}{Br^2}$$
$$= -\frac{\dot\phi^{a^2}}{2A} - \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{4}\left(\phi^a\phi^a - \eta^2\right)^2, \tag{7}$$

$$\frac{3}{4}\frac{\ddot H}{AH} - \frac{3}{4}\frac{\dot A\dot H}{HA^2} - \frac{3}{2}\frac{A'}{ABr} + \frac{3}{r^2H} - \frac{3}{Br^2}$$
$$= \frac{\dot\phi^{a^2}}{2A} + \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{4}\left(\phi^a\phi^a - \eta^2\right)^2, \tag{8}$$

$$\frac{\ddot B}{2AB} - \frac{\dot B^2}{4AB^2} + \frac{\ddot H}{AH^2} - \frac{\dot H^2}{4AH^2} + \frac{A'B'}{4AB}$$
$$-\frac{A''}{2AB} + \frac{A'^2}{4BA^2} - \frac{\dot A\dot B}{4BA^2} - \frac{\dot A\dot H}{2HA^2} - \frac{A'}{rAB}$$
$$-\frac{1}{Br^2} + \frac{\dot B\dot H}{2ABH} + \frac{B'}{rB^2} + \frac{1}{r^2H}$$
$$= \frac{\dot\phi^{a^2}}{2A} - \frac{\phi'^{a^2}}{2B} + \frac{\Lambda}{4}\left(\phi^a\phi^a - \eta^2\right)^2. \tag{9}$$

$$\frac{3}{2}\left(\frac{\dot H}{rH} - \frac{A'\dot H}{2AH} - \frac{\dot B}{rB}\right) = \dot\phi^a\phi'^a. \tag{10}$$

Here '$\cdot$' stands for $\frac{\partial}{\partial t}$ and '$'$' stands for $\frac{\partial}{\partial r}$.

3. Solutions to the field equations

We shall now solve the field equations assuming separable form of the metric coefficients as follows :

$$A = A_1(r)A_2(t); \quad B = B_1(r)B_2(t). \tag{11}$$

Also, we have taken the scalar field triplet in the separable form as

$$\phi^a(r,t) = \phi_1^a(r) + \phi_2^a(t). \tag{12}$$

Putting these separable forms (11) and (12) in (10), we get

$$\left(\frac{\dot H}{rH} - \frac{A_1^1\dot H}{2A_1H}\right) - \frac{\dot B_2}{rB_2} = \frac{2}{3}\dot\phi_2^a\phi_1'^a = 0. \tag{13}$$

Also from (7) and (8), we get

$$\frac{3}{4}\frac{\dot B_2\dot H}{A_2H} + \frac{3}{4}\frac{\dot HB_2}{A_2H} + \frac{3}{4}\frac{\ddot HB_2}{HA_2^2} + \frac{3}{4}\frac{\dot A_2\dot HB_2}{HA_2} - \frac{\dot\phi^{a^2}B_2}{A_2}$$
$$= \frac{3}{2}\frac{B_1^1A_1}{B_1^2r} + \frac{3}{2}\frac{A_1^1}{B_1r} + \frac{\phi'^{a^2}A_1}{B_1} = p, \tag{14}$$

where p is the separation constant.

We shall now solve these equations with the following relations among the metric coefficients :

$$B_2 = B_0H^n \text{ and } A_1 = A_0r^d, \tag{15}$$

where B_0, n, A_0, d are constants.

From (13), we get

$$\frac{3}{2}\left(1 - \frac{d}{2} - n\right)\frac{1}{r\phi'^a} = \frac{\dot\phi_2^a}{\frac{\dot H}{H}} = k, \tag{16}$$

where k is the separation constant.

Solving (16), we get

$$\phi_2^a = k\ln(\phi_{02}^a H) \tag{17}$$

and $$\phi_1^a = b\ln(\phi_{01}^a r), \tag{18}$$

where $b = \frac{3}{2k}\left(1 - \frac{d}{2} - n\right)$ and ϕ_{01}^a and ϕ_{02}^a are integration constants.

From (14), using (15) and (18), we get for space part as

$$\frac{B_1^1}{B_1^2} + \frac{C}{rB_1} = sr^{-d+1}, \tag{19}$$

where $C = d + \frac{2}{3}b^2$; $s = \frac{2p}{3A_0}$.

Solving (19), we get

$$B_l = \frac{1}{Dr^c + \frac{1}{c+d-2} r^{2-d}} \tag{20}$$

where D is an integration constant.

For time part, we introduce an assumption without any loss of generality as

$$A_2 = qH^m, \tag{21}$$

(The value of A_2 different from unity only results in a transformation of time coordinate)

where q, m are constants.

From (19), we get by using (15), (17) and (21)

$$\ddot{H} - l\frac{\dot{H}^2}{H} = vH^{m-n+1}, \tag{22}$$

where $l = n + l + \frac{4}{3}k^2$; $v = \frac{4pq}{3B_0}$,

which has a first integral as

$$\dot{H}^2 = \frac{2v}{m+2-n-2l} H^{m-n+2} + D_l H^{2l}, \tag{23}$$

where D_l is an integration constant.

The integral form of H is

$$\int \frac{dH}{\left[D_l H^{2l} + \frac{2v}{m+2-n-2l} H^{m-n+2}\right]^{\frac{1}{2}}} = \pm(t - t_0) \tag{24}$$

(t_0 = integration constant).

For a different choice of the constants, the solution for H are :

Case I D_l 0

$$H \propto (t - t_0)^{\frac{2}{n-m}} \text{ provided } m \neq n. \tag{25}$$

Case II . $v = 0$

$$H \propto (t - t_0)^{\frac{l}{l-1}} \text{ provided } l \neq 1. \tag{26}$$

Case III : $l = 1$

(i) $m = -1$, $n = -2$

$$H = \frac{2}{v}\frac{e^{\sqrt{D_1}(t-t_0)}}{\left(1 - e^{\sqrt{D_1}(t-t_0)}\right)^2}. \tag{27}$$

(ii) $m = 1$, $n = -1$

$$H = \frac{2\sqrt{\frac{D_1}{v}}\, e^{-2\sqrt{D_1}(t-t_0)}}{1 + e^{-\sqrt{D_1}(t-t_0)}}. \tag{28}$$

Case IV : $l = 2$

(i) $n = 1$, $m = 1$

$$H = \sqrt{\frac{v}{D_1}} \sec\sqrt{v}(t - t_0). \tag{29}$$

(ii) $n = -2$, $m = 1$.

Here, we get

$$-\frac{\sqrt{2vH + D_1}}{D_1 H} - \frac{v}{D_1^{3/2}} \ln\left(\frac{\sqrt{2vH + D_1} - \sqrt{D_1}}{\sqrt{2vH + D_1} + \sqrt{D_1}}\right) = \pm(t - t_0). \tag{30}$$

Case V : $l = 0$

(i) $n = -1$, $m = -1$

$$H = \sqrt{\frac{D_1}{v}} \sin h\sqrt{v}(t - t_0). \tag{31}$$

(ii) $n = -1$, $m = -2$

$$H = \frac{v^2 t^2 - D_1}{2}. \tag{32}$$

Now, the expression for the metric (2) is

$$ds^2 = -A_0 r^d{}_q H^m dt^2 + B_1 B_0 H^n dr^2 + r^2 H d\Omega_3^2. \tag{33}$$

If we define

$$= \sqrt{A_0 q}\int H^{\frac{m-n}{2}} dt$$

and $R = \sqrt{B_l}\int \frac{\sqrt{B_1}}{d}\, dr$

then the above metric can be written as

$$ds^2 = r^d H^n\left[-dT^2 + dR^2 + g(R) f(T)\, d\Omega_3^2\right] \tag{34}$$

$(g(R) = r^{2-d},\ f(T) = H^{1-n})$.

It is to be noted that the metric describes a deficit solid angle which depends on radial and on time coordinates (except for a conformal factor) and hence, it represents a monopole [8, 10] (since monopole exhibits some important properties, particularly in relation to the appearance of nontrivial spacetime topologies [8, 10]).

Again, if we consider $n = 1$, the explicit expression of the metric is (with a proper choice of radial and time coordinates).

$$ds^2 = -dT^2 + dR^2 + g(R)\, d\Omega_3^2 \tag{35}$$

(except for conformal factor).

This solution represents a static model and the solid angle of deficit is a function of radial coordinate only.

Also for $d = 2$, we get the metric of the form

$$ds^2 = -dT^2 + dR^2 + f(T)\, d\Omega_3^2 \tag{36}$$

(except for the conformal factor).

This solution represents a time-dependent model and hence, we get time dependent solid angle of deficit.

However, for $d = 2$, $n = 1$, the above metric (34) is conformally flat and does not describe a monopole.

But it is not possible to find a non-static deficit solid angle in self-similar form as shown by Cho and Vilenkin [11], because we have used a separable form of the metric coefficients.

For future work, it will be interesting to obtain a nonseparable solution for the above field equations.

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